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# A TRANS-IONOSPHERIC SIGNAL SPECIFICATION FOR SATELLITE C<sup>3</sup> APPLICATIONS

Atmospheric Effects Division  
Defense Nuclear Agency  
Washington, D.C. 20305

31 December 1980

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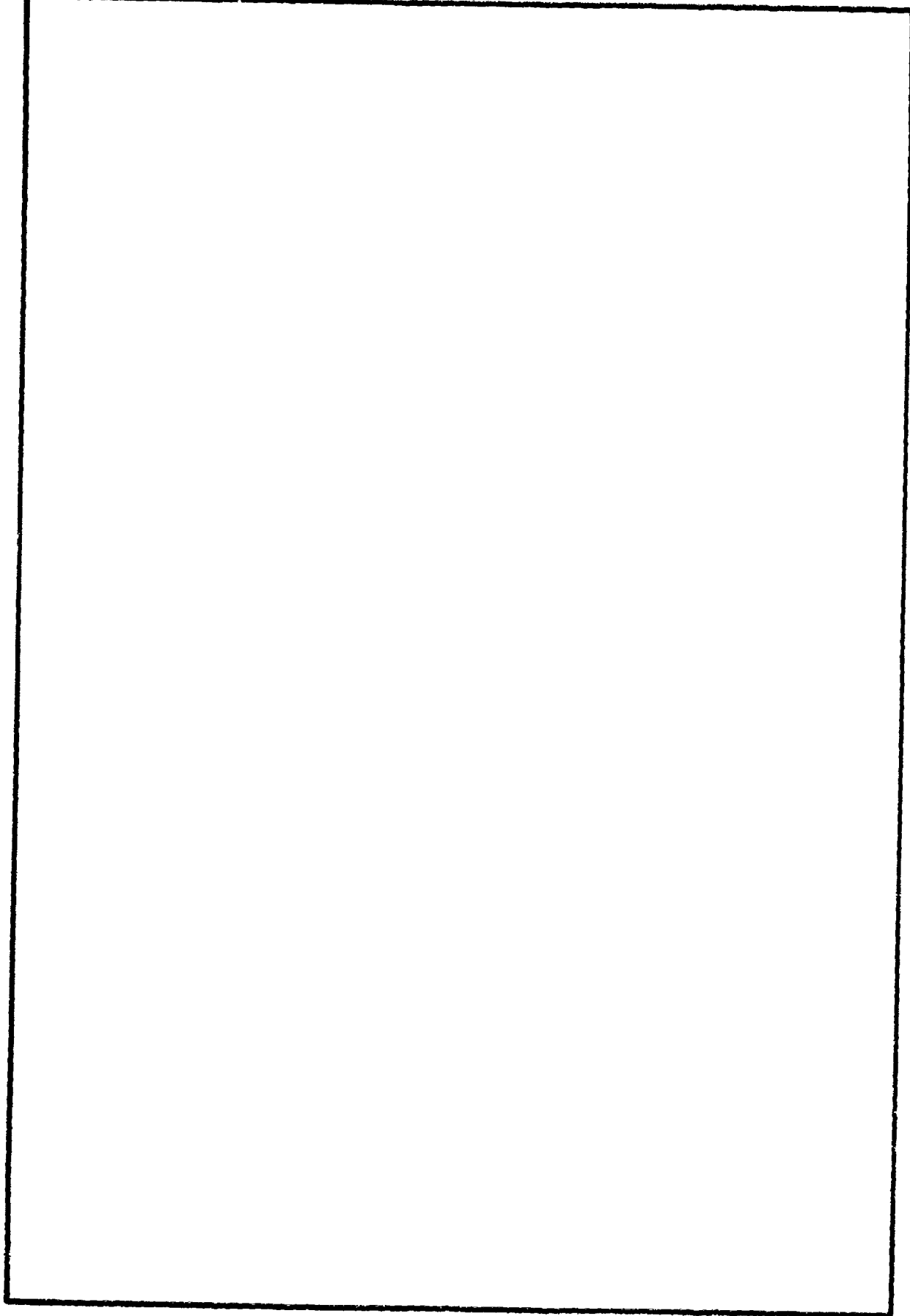
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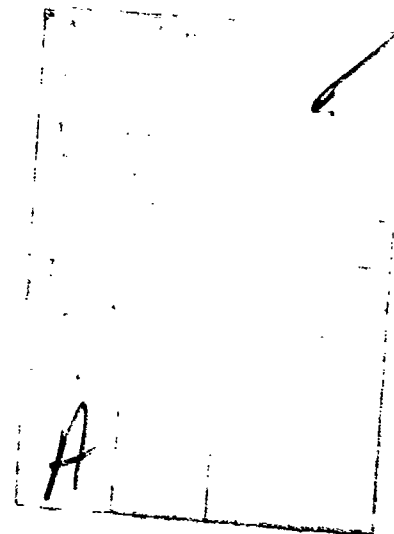


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## PREFACE

The development of this signal structure specification grew out of discussions between the author and Major Nick Alexandrow, then of the Air Force Nuclear Criteria Group Secretariate. I would like to acknowledge his contributions and the assistance of Dr. Dennis Knopp, Mission Research Corporation and Dr. Clifford Prettio, Berkeley Research Associates in preparation of this report.



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## A TRANS-IONOSPHERIC SIGNAL SPECIFICATION FOR SATELLITE C<sup>3</sup> APPLICATIONS

### 1. INTRODUCTION

Proper design of radio frequency systems that must operate through structured or varying plasmas require an accurate and easily implementable signal specification. The specification should be a "reasonable worst case" in that any system which operates with the specified signal can also operate under any likely disturbed condition.

The following specification is the result of a dedicated program sponsored by the Defense Nuclear Agency and the Air Force Weapons Laboratory. Other major contributors were the Naval Research Laboratory and the Air Force Geophysics Laboratory. The program included investigations in the mechanisms that cause structured plasmas, in the propagation of electromagnetic signals through structured plasmas, and in the effects of scintillated or otherwise distorted signals on typical satellite C<sup>3</sup> links. Field experiments and measurement programs were executed to verify theoretical plasma structure and radio propagation predictions and to characterize the morphology of the natural ionosphere. Implicit in the resulting specification are several considerations and assumptions. Rather than discuss all of the supporting logic leading to this specification, a selected bibliography is included. Other supporting information is available in the classified literature. It is noted, however, that this specification should be adequate for all satellite C<sup>3</sup> and data link applications.



## 2. SIGNAL EFFECTS FORMALISM

The effects of a disturbed ionospheric channel are represented by the channel impulse response function.

$$R(t) = \int_0^{\infty} d\tau h(t, \tau) S(t-\tau) \quad (1)$$

$S(t)$  = transmitted signal

$R(t)$  = received signal (complex number representation)

$h(t, \tau)$  = channel impulse response function

The specification channel impulse response function is

$$h(t, \tau) = A(t) \exp \left[ i \frac{2\pi f_c z(t)}{c} - i \frac{e r_0 N(t)}{f_c} \right] \int_{-\infty}^{\infty} d(\Delta f) \tilde{h}_s(t, \Delta f) \exp \left[ -i \frac{e r_0 N(t) \Delta f^2}{f_c^3} - i 2\pi \Delta f \left( \tau - \frac{z(t)}{c} - \frac{e r_0 N(t)}{2\pi f_c^2} \right) \right] \quad (2)$$

where

$f_c$  = carrier frequency (hz)

$c$  = light speed ( $3 \times 10^8$  m/sec)

$z(t)$  = propagation path length (m)

$r_0$  = classical electron radius ( $2.82 \times 10^{-15}$  m)

$N(t)$  = total electron content ( $m^{-2}$ )

For other than circular polarization, Equation 1 should be applied to each polarization state and  $N(t)$  used to calculate the Faraday rotation effects.

$$A(\tau) = \exp(-0.115K_A) \left[ \frac{1}{G(0)} \int_0^\pi d\theta \frac{G(\theta)\theta}{\sigma_\theta^2} \exp(-\theta^2/2\sigma_\theta^2) \right]^{1/2} \quad (3)$$

$K_A$  = absorption (dB)

$G(\theta)$  = antenna gain function

$\sigma_\theta^2$  = energy angle of arrival variance ( $\text{rad}^2$ )

$$\tilde{h}_s(\tau, \Delta f) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} df h_s(f, \tau) \exp(i2\pi\Delta f\tau + i2\pi f\tau) \quad (4)$$

Equation 3 assumes that the transmitter is on the antenna axis and that the antenna pattern is cylindrically symmetric about that axis. If the threat has amplitude fluctuations (always assumed Rayleigh), then  $h_s(f, \tau)$  is a zero mean normally distributed random variable with an autocovariance defined by

$$\overline{h_s^*(f, \tau) h_s(f', \tau')} = \delta(f - f') \delta(\tau - \tau') \Gamma_2(f, \tau) \quad (5a)$$

$$\overline{h_s(f, \tau) h_s(f', \tau')} = 0 \quad (5b)$$

where  $\Gamma_2(f, \tau)$  is the generalized power spectrum and  $\delta(x - x')$  is the Dirac delta function defined for any function  $f(x)$  by

$$\int_{-\infty}^{\infty} dx \delta(x - x') f(x) = f(x')$$

The generalized power spectrum is parameterized by  $\tau_0$ , the scintillated signal decorrelation time, and  $f_0$ , the frequency selective bandwidth. There are two forms for the generalized power spectrum depending on the product  $f_0 T$  where  $T$  is the minimum symbol period. For  $f_0 T \gg 1$

$$\Gamma_2(f, \tau) = \frac{1.864 \tau_0 \delta(\tau)}{[1 + 8.572 \tau_0^2 f^2]^2} \quad (6)$$

This represents the flat fade condition with respect to the symbol period,  $T$ . For  $f_0 T \leq 1$

$$\Gamma_2(f, \tau) = 2^{3/4} \pi^{1/2} \frac{f' \tau}{C_1} \exp \left\{ - \frac{1}{2C_1^2} [(\pi f \tau_0)^2 - 2\pi f' \tau]^2 - (\pi f \tau_0)^2 \right\}$$

$$\int_{-\infty}^{\infty} dx \exp \left\{ -x^4 - 2x^2 \left[ \frac{C_1}{2^{1/2}} \left( 1 + \frac{1}{C_1^2} ((\pi f \tau_0)^2 - 2\pi f' \tau) \right) \right] \right\} \quad (7)$$

where

$$f' = f_0 (1 + C_1^2)^{1/2}$$

$$C_1 = \text{delay parameter } (=0.25)$$

For both equation 6 and 7

$$\int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} d\tau \Gamma_2(f, \tau) = 1 \quad (8)$$

Equation 7 provides for scintillation random delay or, equivalently, frequency selective effects.

If the scintillation threat consists of phase effects only or if there are no scintillations at all, then

$$h_g(f, \tau) = \delta(f) \delta(\tau) \quad (9)$$

If there are scintillations of any kind, then there is a random time varying component in  $N(t)$  in addition to the very large scale (slow) variations.

The total electron content is

$$N(t) = N_L(t) \cdot \int_{-\infty}^{\infty} df g(f) \exp(-i2\pi ft) \quad (10)$$

where  $N_L(t)$  is the large scale (slow) component and  $g(f)$  is a zero mean normally distributed variable representing the random component. The autocovariance of  $g(f)$  is

$$\begin{aligned} g^*(f)g(f') &= \delta(f-f') \frac{\tau_0 (f_c/r_0 c)^2}{[a^2 + (2\pi f \tau_0)^2]^{3/2}} \cdot f \leq f_R \\ &= 0, \quad f > f_R \end{aligned} \quad (11)$$

where

$$g(f) = g^*(-f)$$

$$a^2 = \left( \frac{r_0 c N_L(t)}{f_c} \right)^{-2}$$

$f_R$  = Rayleigh frequency

With the specification of  $K_A$ ,  $\tau_0$ ,  $N(t)$ ,  $\tau_0$ ,  $f_R$ ,  $f_c$ , and  $z(t)$ , we obtain a complete description of the disturbed signal as determined by the propagation environment and system geometry.  $G(\theta)$  is supplied by the user independent of the environment and geometry.

### 3. SIGNAL SPECIFICATION DEFINITION

In principle, a complete signal structure specification would include all of the time/space variations possible for the parameters and functions in Equation 2. In practice, it is sometimes sufficient to specify extremum values or simple functions for the propagation quantities or their derivatives accompanied by application rules. The following table lists a minimum set of specification parameters.

TABLE 1. SIGNAL SPECIFICATION PARAMETERS

SPECIFICATION PARAMETERS
<u>Maximum Values</u>
Absorption, $K_A$
Energy Angle of Arrival Variance, $\sigma_\theta^2$
Transmitter/Receiver Vehicle
Dynamics, $\frac{d^n z}{dt^n}$ , $n=0,3$
Signal Decorrelation Time, $\tau_o$
$\frac{d^n N_L}{dt^n}$ , $n=0,3$
Rayleigh Frequency, $f_R$
<u>Minimum Values</u>
Signal Decorrelation Time, $\tau_o$
Frequency Selective Bandwidth, $f_o$

Two nearly universal application rules apply to  $\tau_0$  and  $f_0$ . For  $\tau_0$ , where both a minimum and maximum is specified, the system must operate over all intermediate values. Similarly, when a minimum  $f_0$  is specified, the system must handle all  $f_0$  from the minimum to the carrier frequency. Exercising these ranges are necessary because the maximum performance degradation may not occur at the extreme values of either  $\tau_0$  or  $f_0$ .

The parameters to be specified are functions of the carrier frequency, the propagation scenario, the link geometries, and the velocities of the system segments. If possible, the specification should cover all of the possible signal variations realized by exercising the above factors over their entire range. Meeting this specification with each link independently would provide the ultimate in survivability, that is, a system that can survive any scenario or circumstance. If this specification cannot be met, then a less severe specification may be possible at some acceptable loss of performance. For example, in nuclear environments, signal specifications are a strong function of the maximum acceptable outage time. Accepting longer outages can often provide significant specification relief. Additional relief might be possible for systems that have multiple links that penetrate the ionosphere at widely dispersed points. Because of the inherent space diversity, the specification would not have to cover the most severe case for any link, but some reduced level of threat. Regardless of the reason, however, any reduction in the specification results in some loss of system applicability.

Methods to calculate the propagation parameters are described in reference 1, excluding absorption, which is covered in reference 2. Calculations based on these methods serve as the basis for developing signal specifications. The first step in the specification development process is to choose the threat environments. For natural environments,

the threat might be either the equatorial or polar ionosphere during the solar maximum depending on the location of the system links. Nuclear threat environments would be calculated by computer codes specifically designed for that purpose from plausible burst yield, altitude, and location combinations. The next step is to find the possible link geometries that maximize the system degradation effects. This usually means minimizing the angle between the link line of sight and the earth's magnetic field where the environment is most severe and maximizing the link path length through the disturbed propagation medium. These two criteria can usually be satisfied simultaneously but, if not, the first almost always takes precedence. The propagation methods in references 1 and 2 are then used to calculate the required propagation quantities as a function of carrier frequency, area coverage, and time. The system segment velocities are chosen to provide maximum and minimum values of the signal decorrelation time. Area coverage is typically represented in contour maps projected on the earth of the propagation quantities which simultaneously show the area coverage for multiple values of those quantities. In ambient environments, contour map sets might be generated for different probabilities of occurrence or different times. The nuclear environment contour sets would correspond to discrete times relative to the times of burst.

These contours, whether ambient or nuclear, provide a data base for the final specification development. The preferred specification would reflect the most severe conditions. If this is not feasible, then the propagation data would permit the necessary trade offs between the threat severity, the applicability and practicality of the system. The final result should be a specification that, if met, would provide adequate system performance at acceptable cost and technical risk.

#### 4. SIGNAL SPECIFICATION IMPLEMENTATION

The disturbed signal effects represented by Equation 2 and the specification parameters are applied twice during a typical system acquisition. First, the signal specification is used during system design to evaluate candidate design solutions. This evaluation is frequently done using computer digital simulation and modeling techniques which provide both simulated degraded signals and detailed descriptions of the dynamic behavior of the candidate designs over all the specified signal conditions rapidly and economically.

The signal specification is also applied during testing and evaluation. At a minimum, the specification determines the range of effects over which the system must be tested. At the other extreme, the specification may define not only the parameter ranges but also the exact form of the signals. This latter circumstance occurs when a system, in its full operational configuration, cannot be exercised in the maximum threat environments. For example, the most severe natural scintillations are coincident with the solar sunspot maximum which occurs only once every eleven years. Systems that must operate in nuclear degraded environments cannot be tested at all because of the 1963 Atmospheric Test Ban Treaty. In these instances, the best remaining method is to degrade the satellite signals artificially by appropriately designed link simulators. By placing a simulator at one or more points in the operational system, testing can be done limited only by the accuracy of and confidence in the signal specification.

When link simulators are necessary for testing and evaluation, questions arise over what constitutes a necessary and sufficient test program. For example, is it necessary to include all of the effects represented by Equation 2 in every test simultaneously? Is it necessary



to test simultaneously every link in a system or even every link individually? How do you test a system to include the responses of the operators as they react to degradation on system links from propagation or other simultaneous threats? On a longer term, is repeated testing necessary to insure that the system hardware, the operational procedures, and the operator responses remain adequate to maintain the required performance? These and other similar issues need consideration for each system tested. The answers to these questions, broadly known as "compliance standards," should accompany the signal specification for each system to provide a completely defined test and evaluation program.

## 5. SUMMARY

The preceding sections have detailed the form of a signal structure specification for application to satellite C<sup>3</sup> systems. The specification is intended as a design tool as well as a definition of the signal conditions in which the system must operate. Adequate techniques exist to apply the specification both in computer simulation during design and by a link simulator for test and evaluation.

Appendix A describes methods to implement specific realizations of the channel impulse response function. Appendix D provides a brief description of link simulators.

## REFERENCES

1. Wittwer, Leon A., Radio Wave Propagation in Structured Ionization for Satellite Applications, DNA 5304D, (Contains a summary of the techniques used to calculate signal structure parameters in disturbed natural or nuclear propagation environments and the development of the generalized power spectrum.)
2. Knapp, W. S. and Schwartz, K., Aids for the Study of Electromagnetic Blackout, DNA 3499H, 25 Feb 1975. (Compendium of selected graphs, charts, equations, and relations useful in the analysis of electromagnetic blackout caused by nuclear explosions.)
3. Knepp, D. L., Multiple Phase-Screen Propagation Analysis for Defense Satellite Communications System, DNA 4424T, (MRC-R-332), Mission Research Corporation, September 1977. (Describes a numerical propagation simulation technique in detail, and presents scintillation calculations for X-band (7.5 GHz). Also see the following three reports for additional multiple phase screen propagation calculation results.)
4. Wittwer, L. A., The Propagation of Satellite Signals Through Turbulent Media, AFWL-TR-77-183, Air Force Weapons Laboratory, January 1978. (Comprehensive treatment of effects of striated ionospheres on satellite signals. Calculations include effects of various striation power spectral densities and scale sizes.)
5. Wittwer, L. A., et al., UHF Propagation Effects in Scintillated Environments, AFWL-TR-76-304, Air Force Weapons Laboratory, August 1977. (Describes propagation calculational techniques and presents results for UHF satellite signals. Demonstrates that Rayleigh signal statistics are a reasonable worst case representation.)
6. Hendrick, R. W., Propagation of Microwave Satellite Signals Through Striated Media, DNA 4412T, (MRC-R-334), Mission Research Corporation, September 1977. (Analyzes results of multiple phase screen propagation calculations over a frequency range from 300 MHz to 8 GHz.)

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# APPENDIX A THE GENERATION OF DIGITAL REALIZATIONS OF THE CHANNEL IMPULSE RESPONSE FUNCTION

The channel impulse response function is

$$h(t, \tau) = A(t) \exp \left[ i \frac{2\pi f_c z(t)}{c} - i \frac{cr_0 N(t)}{f_c} \right] \int_{-\infty}^{\infty} d(\Delta f) \tilde{h}_s(t, \Delta f) \exp \left[ -i \frac{cr_0 N(t) \Delta f^2}{f_c^3} - i 2\pi \Delta f \left( \tau - \frac{z(t)}{c} - \frac{cr_0 N(t)}{2\pi f_c^2} \right) \right] \quad (A-1)$$

where

$A(t)$  = absorption and antenna loss

$f_c$  = carrier frequency

$c$  = light speed

$z(t)$  = propagation path length

$r_0$  = classical electron radius

$N(t) = N_L(t) + N_R(t)$

$N_L(t)$  = slow trend total electron content

$N_R(t)$  = random total electron content

The generation of digital realizations of Equation A-1, in general, has three steps. First,  $A(t)$ ,  $N_L(t)$ , and  $z(t)$  are discretized. Next, samples of  $\tilde{h}_s(t, \Delta f)$  and  $N_R(t)$  are calculated using Monte Carlo techniques. Finally, the integral over  $\Delta f$  is evaluated. Most of the complexity in generating these digital representations lies in the random sampling of  $\tilde{h}_s(t, \Delta f)$  and  $N_R(t)$ . Most of this appendix will concentrate

on these sampling techniques.

The statistical properties of  $\hat{h}_s(t, \Delta f)$  are defined by

$$\hat{h}_s(t, \Delta f) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} df h_s(f, \tau) \exp(i2\pi\Delta f\tau + i2\pi ft) \quad (A-2)$$

where  $h_s(f, \tau)$  is a zero mean normally distributed random variable with its autocovariance defined by

$$\overline{h_s^*(f, \tau) h_s(f', \tau')} = \delta(f - f') \delta(\tau - \tau') \Gamma_2(f, \tau) \quad (A-3a)$$

$$\overline{h_s(f, \tau) h_s(f', \tau')} = 0 \quad (A-3b)$$

where  $\delta(f)$  = Dirac delta function

$\Gamma_2(f, \tau)$  = generalized power spectrum

There are two distinct cases to be considered. First, for flat fading defined by  $f_0 T \gg 1$  where  $f_0$  is the frequency selective bandwidth and  $T$  is the symbol period, the generalized power spectrum is

$$\Gamma_2(f, \tau) = \frac{1.864 \tau_0^{-1} \delta(\tau)}{\left[1 + 8.572 (f \tau_0)^2\right]^2} \quad (A-4)$$

where  $\tau_0$  is the signal decorrelation time. Because of the delta correlation in delay,  $\hat{h}_s(t, \Delta f)$  is not a function of  $\Delta f$  and the problem is reduced to generating a sequence that is only a function of time.

For any  $\Delta f$ , let

$$\hat{h}_s(t_i, \Delta f) = \rho \hat{h}_s(t_{i-1}, \Delta f) + (1 - \rho^2)^{1/2} u_i \quad (A-5)$$

$$u_i = \rho u_{i-1} + (1 - \rho^2)^{1/2} g_i \quad (A-6)$$

where

$$\rho = \exp(-2.146 \Delta t / \tau_0)$$

$$\Delta t = t_i - t_{i-1}$$

$g_i$  = zero mean normally distributed complex sample

$$g_i^* g_i = \tanh(2.146 \Delta t / \tau_0)$$

The sequence can be initiated by choosing an initial value for  $\hat{h}_s(t_i, \Delta f)$  of order unity and a value for  $u_i$  of order  $\overline{g_i^* g_i}^{1/2}$ . Appendix B describes a simple algorithm for sampling zero mean normally distributed complex variables. The samples from Equation A-5 represent instantaneous samples and no filtering is implied. Also, the following conditions should be met.

$$t_0 / \Delta t \geq 10 \quad (A-7a)$$

$$M \geq 100 \tau_0 / \Delta \tau \quad (A-7b)$$

where  $M$  is the minimum number of points in any sequence generated by Equation A-5.

The second case, defined by  $f_0 T \leq 1$ , is frequency selective fading. Equation A-2, in finite difference form, is

$$\tilde{h}_s(t_i, \Delta f_k) = \Delta f' \sum_{n=-M}^M \Delta \tau \sum_j h_s(f_n, \tau_j) \exp(i2\pi f_n t_i + i2\pi \Delta f_k \tau_j) \quad (A-8)$$

where

$$\Delta f' = 1/(2M\Delta t)$$

$$\Delta t = t_i - t_{i-1}$$

$$f_n = n\Delta f'$$

$$t_i = i\Delta t, \quad |i| \leq M$$

$$|\Delta f_k| \leq 1/(2\Delta \tau)$$

From Equations A-3a and A-3b, the statistics of  $h_s(f_n, \tau_j)$  can be written as

$$\overline{h_s^*(f_n, \tau_j) h_s(f_m, \tau_i)} = \delta_{mn} \delta_{ij} \left( \frac{1}{\Delta f' \Delta \tau} \right)^2 \int_{\tau_j - \Delta \tau/2}^{\tau_j + \Delta \tau/2} d\tau \int_{f_n - \Delta f'/2}^{f_n + \Delta f'/2} df \quad r_2(f, \tau) \quad (A-9a)$$

$$\overline{h_s^*(f_n, \tau_j) h_s(f_m, \tau_i)} = 0 \quad (A-9b)$$

$$\overline{h_s^*(f_n, \tau_i)} = 0 \quad (A-9c)$$

where, for  $f_0 T \leq 1$ ,

$$r_2(f, \tau) = 2^{3/4} \pi^{1/2} \frac{f' \tau_0}{C_1^{1/2}} \exp \left\{ -\frac{1}{2C_1^2} \left[ (\pi \tau_0 f)^2 - 2\pi f' \tau \right]^2 - (\pi \tau_0 f)^2 \right\} \\ \int_{-\infty}^{\infty} dx \exp \left\{ -x^4 - 2x^2 \left[ \frac{C_1}{2^{1/2}} \left( 1 + \frac{1}{C_1^2} \left( (\pi \tau_0 f)^2 - 2\pi f' \tau \right) \right) \right] \right\} \quad (A-10)$$

$$f' = f_0 (1 + C_1^2)^{1/2}$$

$C_1$  = delay parameter ( $\approx 0.25$ )

Equation A-10 and related quantities are evaluated in Appendix C. Generating the  $\tilde{h}_s(t_1, \Delta f_k)$  consists of evaluating the  $h_s(f_n, \tau_j)$  using Equations A-9, A-10, and Appendix B and then evaluating Equation A-8. For adequate statistical sampling and numerical resolution, it is necessary that

$$\tau_0 / \Delta t > 10 \quad (A-11a)$$

$$M > 50 \tau_0 / \Delta t \quad (A-11b)$$

$$\Delta \tau \leq T/5 \quad (A-11c)$$

$$\Delta(\Delta f) \leq 1 / \max(1.1/f_0, 10T) \quad (A-11d)$$

$$|\tau_0 f| \leq 0.75 \quad (A-11e)$$

$$-0.25 \leq 2\pi f_0 \tau \leq 3.45 \quad (A-11f)$$

$$|\tau| \leq 5T \quad (A-11g)$$



where  $(\Delta f) = \Delta f_k - \Delta f_{k-1}$ . Condition A-11b is a bare minimum. Larger M or multiple sequences are advisable. Equations A-11e and A-11f prescribe the ranges of  $\tau$  and  $f$  in Equation A-10 that include 95 percent of the total energy. The coefficients,  $h_s(f_n, \tau_j)$ , are not necessary outside those ranges. Equation A-11g and the T dependent term in Equation A-11d reflect an estimate of the required delay range to handle dispersion. The integral in Equation A-9 can be evaluated to one percent with trapezoidal integration with frequency and delay increments less than or equal to  $0.03/\tau_0$  and  $0.03/f_0$ , respectively.

The representation used for the random portion of the total electron content depends on the application. The first use of  $N_R(t)$  in Equation A-1 is in the carrier phase where it can degrade phase or frequency acquisition and tracking. For phase effects

$$N_R(t_i) = \Delta f'' \sum_{n=-M'}^{M'} g(f'_n) \exp(i 2\pi f'_n t_i) \quad (A-12)$$

where  $\Delta f'' = 1/(2M' \Delta \tau')$

$$\Delta \tau' = t_i - t_{i-1}$$

$$f'_n = n \Delta f''$$

$$t_i = i \Delta \tau', |i| \leq M'$$

$g(f'_n)$  is a zero mean normally distributed variable whose remaining statistics are defined by

$$\frac{g^*(f'_n)g(f'_m)}{(\Delta f'')^2} = \frac{\delta_{nm}}{(\Delta f'')^2} \int_{f'_n - \Delta f''/2}^{f'_n + \Delta f''/2} df$$

$$\frac{\tau_0 (f_c / r_0 c)^2}{[a^2 + (2\pi f \tau_0)^2]^{3/2}}, \quad |f'_n| \leq f_R \quad (\text{A-13a})$$

$$= 0, \quad |f'_n| > f_R \quad (\text{A-13b})$$

where

$$g(f'_{-n}) = g^*(f'_n)$$

$$a^2 = \left( \frac{r_0 c v_L(t)}{f_c} \right)^2$$

$f_R$  = Rayleigh frequency

Initial estimates for  $\Delta t'$  and  $M'$  are

$$\Delta t' \leq 1/(4f_R) \quad (\text{A-14a})$$

$$M' \geq 512 \quad (\text{A-14b})$$

For frequency tracking, Equation A-12 cannot be used. Instead, let

$$N_R(t_i) = \sum_{n=-M'}^{M'} \frac{1}{f_R} g(f'_n) \exp(i 2\pi f'_n t_i) \quad (\text{A-15})$$

Also,

$$\frac{g^*(f'_n)g(f'_m)}{(\Delta f_n)^2} = \frac{\delta_{mn}}{(\Delta f_n)^2} \int_{|f'_n|/K}^{|f'_n|K} df \frac{\tau_o (f_c/r_o c)^2}{[a^2 + (2\pi f \tau_o)^2]^{3/2}}, \quad |f'_n| \leq f_R, n \neq 0 \quad (A-16a)$$

$$= \frac{\delta_{m0}}{(\Delta f_o)^2} \int_{-\Delta f_o/2}^{\Delta f_o/2} df \frac{\tau_o (f_c/r_o c)^2}{[a^2 + (2\pi f \tau_o)^2]^{3/2}}, \quad |f'_n| \leq f_R, n=0 \quad (A-16b)$$

$$= 0, \quad |f'_n| > f_R \quad (A-16c)$$

where

$$g(f'_{-n}) = g^*(f'_n)$$

$$f'_o = 0$$

$$f'_n = \text{sign}(n) \left( \frac{a}{100\tau_o} \right) (K^2)^{n-1}, \quad n \neq 0$$

$$\Delta f_o = 2f'_1/K$$

$$\Delta f_n = |f'_n| (K-1/K), \quad n \neq 0$$

$$K = (f_R/f_1)^{1/M'}$$

$$t_{i+1} - t_i \leq 1/(2f_R)$$

$$M' \geq 512$$

$$|t_i| \leq 1/(2f'_1)$$

The second use of  $N(t)$  in Equation A-1 is in the dispersive phase.  $N_R(t)$  can usually be omitted because the dispersive effects are primarily determined by the magnitude of  $N(t)$  and not the time dependence.

The last use of  $N_R(t)$  determines the group delay and group delay rate which can degrade time synchronization if sufficiently large. Equations A-15 and A-16 should be used with  $f_R$  replaced by  $f_T$ .

$$f_T = f_R / \left[ 1 + (10 f_c f_R \tau_o T)^2 \right]^{1/2} \quad (A-17)$$

If simultaneous handling of the total electron content effects is necessary, then Equations A-15 and A-16 are necessary. The substitution of  $f_T$  should not be used. Unfortunately, the number of time increments necessary to span the time range can be very large. Exercising the range of time may not be necessary in all cases. Some experimentation is often indicated. Another problem with Equation A-15 is that the sum cannot be evaluated with fast fourier transforms. Finally, multiple sequences of  $N_R(t_i)$  should be used to test the adequacy of the sampling for either Equation A-12 or A-15.

The entire channel impulse response function is

$$h(t_i, \tau_j) = A(t_i) \exp \left[ i \frac{2\pi f_c z(t_i)}{c} - i \frac{c r_o N(t_i)}{f_c} \right] \Delta(\Delta f) \sum_{k=-L}^L \tilde{h}_s(t_i, \Delta f_k) \exp \left[ -i \frac{c r_o N(t_i) \Delta f_k^2}{f_c^3} - i 2\pi \Delta f_k \left( \tau_j - \frac{z(t_i)}{c} - \frac{c r_o N(t_i)}{2\pi f_c^2} \right) \right] \quad (A-18)$$

where

$$L=1/(2\Delta\tau\Delta(\Delta f))$$

$$\left| \tau_j - \frac{z(t_i)}{c} - \frac{cr_0 N(t_i)}{2\pi f_c^2} \right| \leq L\Delta\tau$$

In principle, one only needs to insert the proper quantities discretized to the smallest increment in time, delay, or  $\Delta f_k$  and sum over  $\Delta f_k$ . In practice, however, the wide range of the delay and time necessary to adequately sample the various functions make this impractical. It is usually necessary to handle some of the terms independently, particularly the total electron content and path length determined effects. The channel impulse function is thereby greatly simplified.

$$h(t_i, \tau_j) = A(t_i) h_s(t_i, \tau_j) \quad (A-19)$$

where for  $f_0 T \leq 1$

$$h_s(t_i, \tau_j) = \Delta f \sum_{n=-M}^M h_s(f_n, \tau_j) \exp(i2\pi f_n t_i)$$

or for  $f_0 T > 1$

$$h_s(t_i, \tau_j) \approx \hat{h}_s(t_i, \Delta f), \quad \tau_j = 0$$

$$\approx 0, \quad \tau_j \neq 0$$

A fortran program for generating  $h_s(t_i, \tau_j)$  can be found in Appendix E. Equation A-19 is implementable in a link simulator as described in Appendix D.

An important consideration for implementing Equation A-18 or A-19 is the dynamic range required to resolve the amplitude fluctuations, if present. The amplitude of the channel impulse response function at each discrete delay is a Rayleigh distributed variable. Let the amplitude of

$h(t_i, \tau_j)$  be  $R_{ij}$ . The distribution function for  $R_{ij}$  is

$$P(R_{ij}) = \frac{2R_{ij}}{\sigma_j^2} \exp\left(-R_{ij}^2/\sigma_j^2\right) \quad (A-20)$$

where

$$\sigma_j^2 = \int_{-\infty}^{\infty} df \int_{\tau_j - \Delta\tau/2}^{\tau_j + \Delta\tau/2} d\tau \Gamma_2(f, \tau)$$

Equation A-20 is easily invertable to form probability statements about  $R_{ij}$ . Thus,

$$1 - \exp\left(-\frac{R_{ij}^2}{\sigma_j^2}\right) = \text{probability that } R_{ij} \leq R_{ij}'$$

$$\exp\left(-\frac{R_{ij}^2}{\sigma_j^2}\right) = \text{probability that } R_{ij} \geq R_{ij}'$$

We require that the amplitude not exceed the dynamic range 99.8 percent of the time. The upper and lower bounds on the amplitude are, respectively

$$(R_{ij}/\sigma_j)_u = \left[-\ln(0.001)\right]^{1/2} = 2.63$$

$$(R_{ij}/\sigma_j)_l = \left[-\ln(0.999)\right]^{1/2} = 3.16 \times 10^{-2}$$

where the probability that the amplitude exceeds the dynamic range, 0.002, is equally split between the limits. In decibels

$$-30\text{dB} \leq 20 \log_{10} \left( \frac{R_{ij}}{\sigma_j} \right) \leq 8.4\text{dB}$$

Thus, 38.4 dB of dynamic range is required. If this is not feasible, then the loss of accuracy should be at the largest amplitudes. This is because system performance is much more sensitive to amplitude fades than to amplitude enhancements.

The resolution in the amplitude and phase is a function of the resolution of the real and imaginary components of  $h(t_i, \tau_j)$ . The poorest resolution occurs in deep fades at the smallest amplitude. The maximum increment is defined by requiring that successive values of either component of  $h(t_i, \tau_j)$  not vary by more than ten percent. This gives adequate accuracy on the amplitude and resolves the phase to within six degrees. If the component increment is fixed, then the smallest value must be used. The fixed increment for either component can be expressed as one tenth of  $(R_{ij}/\sigma_j)_1$ . Variable increments are more desirable because they permit more efficient storage of component data. They also permit the use of logarithmic digital to analog converters in link simulators as described in Appendix D.

The total electron content effects neglected in Equation A-19 can be bounded to determine necessary but not sufficient conditions for successful system operation. The total electron content and its derivatives are gaussian distributed random variables with the following means and standard deviations.

$$\overline{\frac{d^m N(t)}{dt^m}} = \frac{d^m N_L(t)}{dt^m} \quad (A-21a)$$

$$\sigma_{Nm} = \left[ 2 \int_0^{f_R} df \frac{(2\pi f)^{2m} \tau_0 (f_c/r_0 c)^2}{[a^2 + (2\pi f \tau_0)^2]^{3/2}} \right]^{1/2} \quad (A-21b)$$

These statistical moments, when converted to moments of phase, doppler, doppler rate, jerk, etc., allow performance degradation estimates. These statistics also determine the amount of time that any of the derivatives exceed some value. As such, they provide the means for specifying various system parameters consistent with some performance requirement.

The total electron content drives the dispersive distortion of the signal waveform. As a measure of this distortion, we use the expectation value of the absolute value of the delay coordinate.

$$\overline{|\tau|} = \int_{-\infty}^{\infty} d\tau |\tau| \int_{-\infty}^{\infty} d(\Delta f) \exp \left[ -i \frac{c r_0 N(t) \Delta f^2}{f_c^3} - i 2\pi \Delta f \tau \right] \quad (A-22a)$$

$$= \frac{c r_0 N(t)}{\pi^2 f_c^3} \quad (A-22b)$$

Since the statistical moments of the measure of distortion are proportional to the moments of the total electron content, performance criteria can be developed as a function of modulation and other system details. The criteria should guarantee that the ratio of  $\overline{|\tau|}$  to T be small.



The finite difference equations have been written in the lowest order. This was done for simplicity and to be consistent with fast fourier transform algorithms. These algorithms can evaluate Equations A-8, A-12, A-18, and A-19 much more efficiently than conventional methods.

APPENDIX B  
A RANDOM NUMBER GENERATOR FOR  
NORMALLY DISTRIBUTED COMPLEX NUMBERS

Let  $g$  be a normally distributed complex number defined by

$$\overline{g^*g} = \sigma^2 \quad (\text{B-1a})$$

$$\overline{gg} = 0 \quad (\text{B-1b})$$

$$\overline{g} = 0 \quad (\text{B-1c})$$

$$P(|g|) = \frac{2|g|}{\sigma^2} \exp(-|g|^2/\sigma^2) \quad (\text{B-1d})$$

where  $P(|g|)$  is the probability distribution for the amplitude of  $g$ . Let  $R_i$  be a uniformly distributed random variable on the interval,  $0 \leq R_i \leq 1$ . A sample of  $g$  is calculated by

$$g = \sigma \left[ -\ln(R_i) \right]^{1/2} \exp(i2\pi R_{i+1}) \quad (\text{B-2})$$

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# APPENDIX C EVALUATION OF THE GENERALIZED, DELAY, AND FREQUENCY POWER SPECTRUMS FOR $f_0 T_0 \leq 1$

The generalized power spectrum is

$$P_2(f, \tau) = 2^{3/4} \pi^{1/2} \frac{f_0^2 \tau_0}{C_1^{1/2}} \exp \left\{ -\frac{1}{2C_1^2} \left[ (\pi \tau_0 f)^2 - 2\pi f_0 \tau \right]^2 - (\pi \tau_0 f)^2 \right\} \int_{-\infty}^{\infty} dx \exp \left\{ -x^4 - 2x^2 \left[ \frac{C_1}{2^{1/2}} \left( 1 + \frac{1}{2} \frac{(\pi \tau_0 f)^2 - 2\pi f_0 \tau}{C_1^2} \right) \right] \right\} \quad (C-1)$$

where

$$f_0 = f_0 (1 + C_1^2)^{1/2}$$

$$C_1 = \text{delay parameter } (=0.25)$$

$$f_0 = \text{frequency selective bandwidth}$$

$$\tau_0 = \text{signal decorrelation time}$$

A fit to Equation C-1, good to one percent, is

$$P_2(f, \tau) = 2.981 \frac{f_0^2 \tau_0}{C_1^{1/2}} \exp \left( \frac{C_1^2}{2} - 2\pi f_0 \tau \right)$$

$$C \left[ \frac{C_1}{2^{1/2}} + \frac{1}{2} \frac{(\pi \tau_0 f)^2 - 2\pi f_0 \tau}{C_1^2} \right] \quad (C-2)$$

where

$$G(z) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{\exp(-z^2)}{z^{\frac{1}{2}}} \left(1 - \frac{0.191}{z^2} + \frac{0.197}{z^4} - \frac{0.112}{z^6}\right), \quad z \geq 1$$

$$G(z) = 1.813 \exp(-0.675z - 0.729z^2 - 0.109z^3 + 0.031z^4), \quad -1 \leq z \leq 1$$

$$G(z) = \frac{\pi^{\frac{1}{2}}}{|z|^{\frac{1}{2}}} \left(1 + \frac{0.290}{z^2} - \frac{0.178}{z^4} + \frac{0.0014}{z^6}\right), \quad z < -1$$

The delay power spectrum is defined as

$$\Gamma_2(\tau) = \int_{-\infty}^{\infty} df \Gamma_2(f, \tau) \quad (C-3)$$

$$\Gamma_2(\tau) = \pi f_1^2 \exp \left[ \frac{1}{2} \left( \frac{c_1}{f_1} - \frac{2\pi f_1 \tau}{c_1} \right)^2 - \frac{1}{2} \left( \frac{2\pi f_1 \tau}{c_1} \right)^2 \right] \left\{ 1 - \exp \left[ \frac{1}{2} \left( \frac{c_1}{f_1} - \frac{2\pi f_1 \tau}{c_1} \right) \right] \right\} \quad (C-4)$$

Let

$$u = \frac{1}{2} \left( \frac{c_1}{f_1} - \frac{2\pi f_1 \tau}{c_1} \right)$$

$$\tau = (1 + 0.47647 \left| \frac{u}{c_1} \right|)^{-1}$$

Then

$$\Gamma_2(\tau) = \pi f' \exp\left[-\frac{(2\pi f' \tau)^2}{2C_1^2}\right] F(t) , \quad z \geq 0 \quad (C-5a)$$

$$\Gamma_2(\tau) = \pi f' \exp\left[\frac{C_1^2}{2} - 2\pi f' \tau\right] \left[2 - F(t) \exp(-z^2)\right] , \quad z < 0 \quad (C-5b)$$

where  $F(t) = \left((0.7478556t - 0.0958798)t + 0.3480242\right)t$

Equation C-5 is accurate to about five figures. The frequency power spectrum is defined as

$$\bar{\Gamma}_2(f) = \int_{-\infty}^{\infty} d\tau \Gamma_2(f, \tau) \quad (C-6)$$

$$\Gamma_2(f) = \pi^{1/2} \tau_0 \exp\left[-(\pi \tau_0 f)^2\right] \quad (C-7)$$

The normalization integrals for the above spectrums are

$$\int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} df \Gamma_2(f, \tau) = 1 \quad (C-8)$$

$$\int_{-\infty}^{\infty} d\tau \bar{\Gamma}_2(\tau) = 1 \quad (C-9)$$

$$\int_{-\infty}^{\infty} df \bar{\Gamma}_2(f) = 1 \quad (C-10)$$

9  
Frequently, integrals over the various power spectrums are necessary as, for example, in Equation A-9. Trapezoidal integration with the following integration increments is accurate to approximately one percent.

$$\Delta f_1 \leq 0.03/\tau_0 \quad (C-11a)$$

$$\Delta \tau_1 \leq 0.03/f_0 \quad (C-11b)$$

## APPENDIX D PROPAGATION EFFECTS LINK SIMULATORS

Figure D-1 shows a general implementation of a link simulator which is the hardware analog of the channel impulse response function defined by

$$R(t) = \int_0^t dt h(t, \tau) S(t - \tau) \quad (D-1)$$

where

- $S(t)$  = transmitted (input) signal
- $R(t)$  = received (output) signal
- $h(t, \tau)$  = channel impulse response function

The link simulator represents a discretized version of D-1 where

$$I_n(t) = D/A \left\{ \text{Real} \left[ h(t_i, t_n) \Delta \tau \right] \right\} \quad (D-2a)$$

$$Q_n(t) = D/A \left\{ \text{Imag} \left[ h(t_i, t_n) \Delta \tau \right] \right\} \quad (D-2b)$$

$D/A \{ \}$  = digital to analog conversion

Algorithms to calculate values of the channel impulse response function were detailed in Appendix A. The sampling requirements in Appendix A



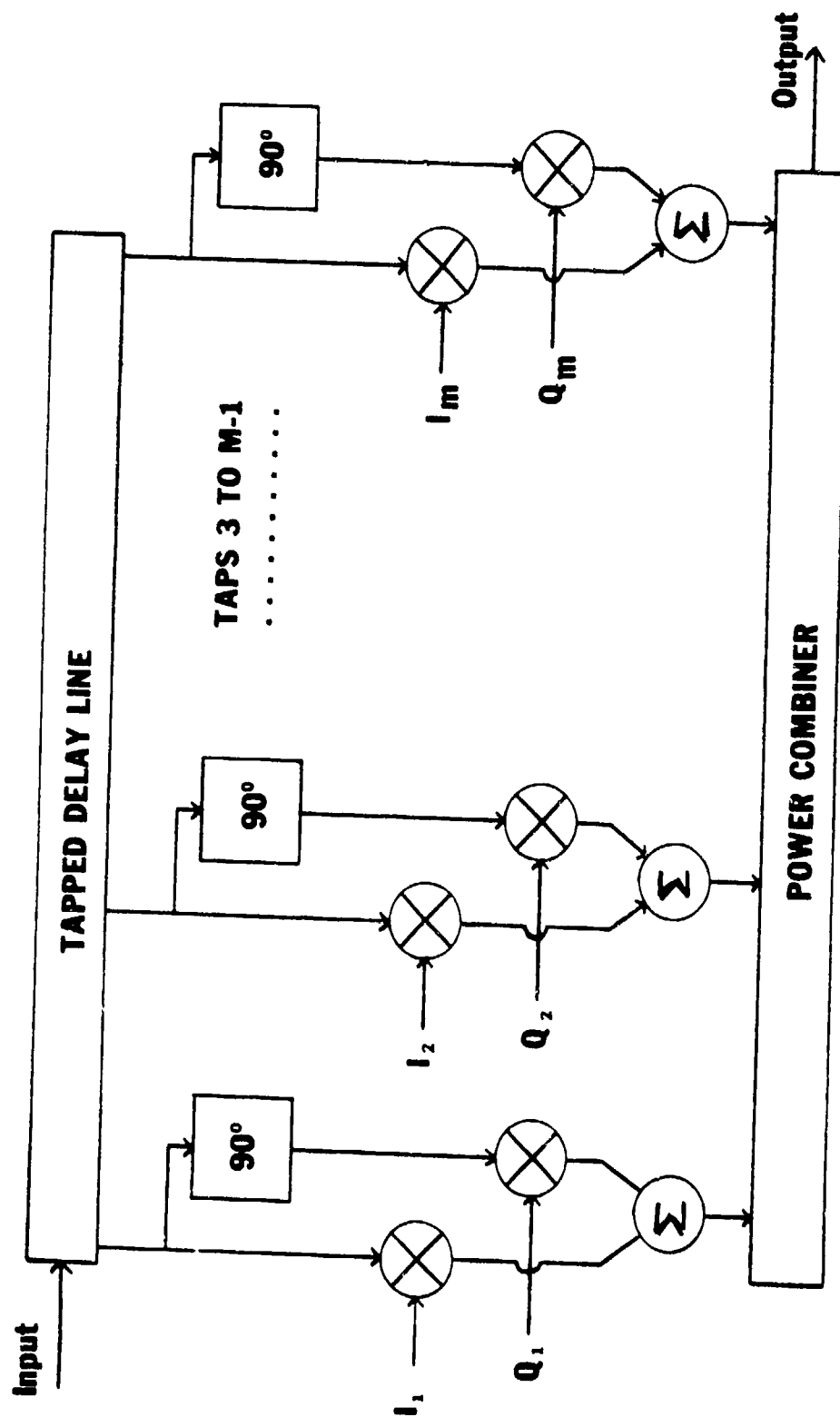


Figure D-1. Propagation Effects Simulator

are sufficient to support the digital to analog operation without any additional filtering to suppress aliasing.

Figure D-2 shows a particularly simple link simulator for flat fading and when all other effects are ignored or simulated separately.

The delay is used to get an independent noise source for the I channel without an extra noise generator. The filters are single pole providing in cascade the power spectrum represented by Equation 6. For a RC filter, the time constant is

$$RC = \tau_o / 2.146 \quad (D-3)$$

The input/output can be at any frequency level.

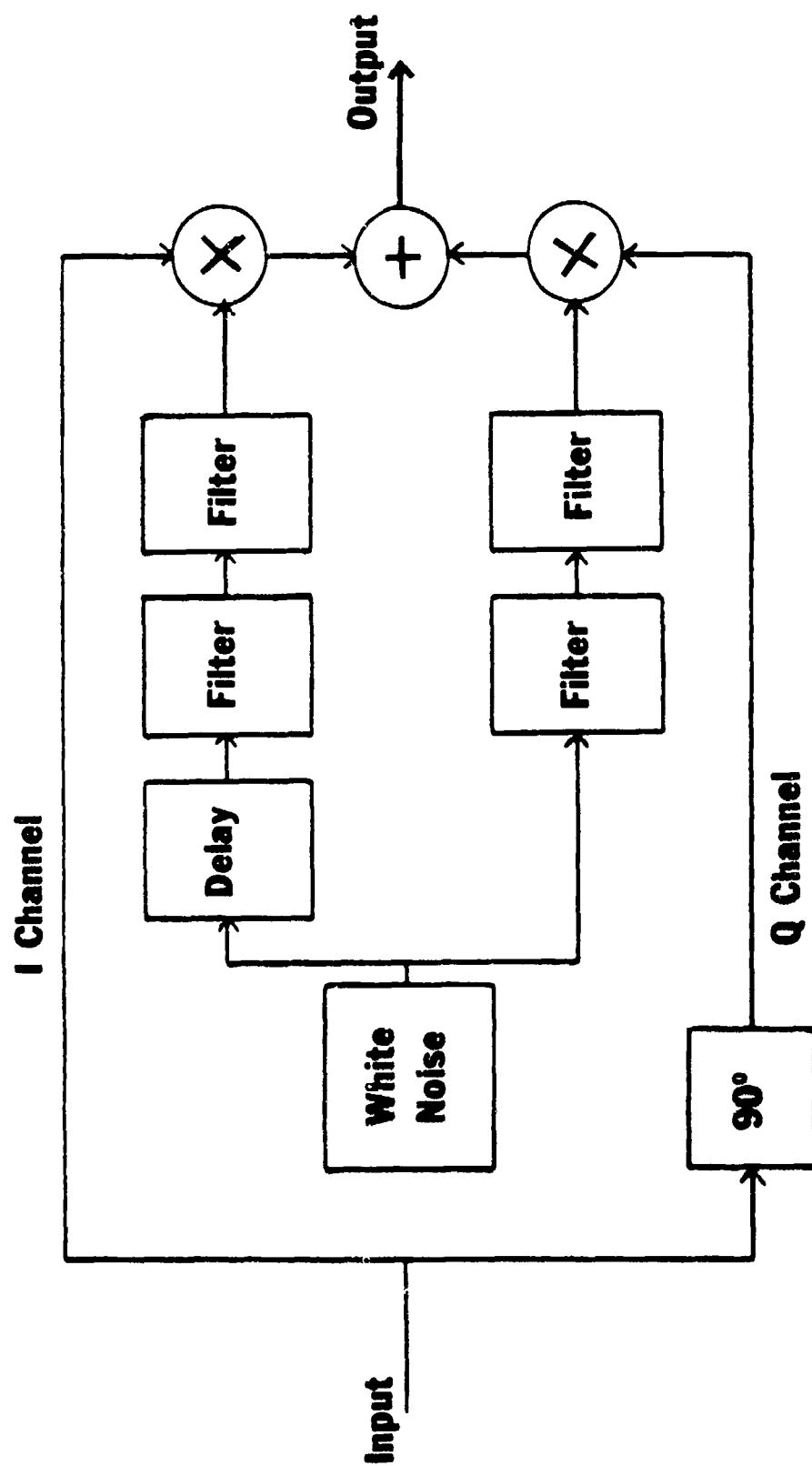


Figure D-2. Flat Fade Simulator

APPENDIX E  
CIRF, A FORTRAN PROGRAM FOR GENERATING SAMPLE DIGITAL  
REPRESENTATIONS OF THE CHANNEL IMPULSE RESPONSE FUNCTION

This appendix contains a fortran program to sample digital representations of the channel impulse response function in Equation A-19 less the absorption and antenna loss terms. The coding is consistent with Fortran IV with the exception of the dimension declaration in the fast fourier transform subroutines (see comment in SUBROUTINE SETD list).

The methods used to generate the frequency selective sequences are also used for the flat fade sequences rather than Equations A-5 and A-6. The flat fade sequence generation is thus slower, but the sequences generated are continuous from the last sequence term back around to the first term, a very useful property for hardware testing applications. The flat fade sequences are nevertheless consistent with Equation A-4.

The program tests the input for compliance with Equations A-11a, A-11b, A-11e, and A-11f. The recommended integration increments are also enforced. The use of floating point variables for the channel impulse response function output guarantees compliance with the stated phase and amplitude resolution requirements on all known computer systems that support Fortran IV. The user is responsible for satisfying Equation A-11c.

The code provides time sequences of the impulse response function. The first sequence centers on a delay of  $-0.04/F0 + DD/2$  where  $F0$  is the frequency selective bandwidth and  $DD$  is the delay increment. The main program loop over delay ends at statement label 70. Just prior to that statement, the time sequence for the current delay is in the H array.  $H(I+1-1)$  and  $H(I+1)$  represent the in-phase (real) and quadrature (imaginary) impulse response function components, respectively. The user is

responsible for providing the extra coding for disposing of the sequence data. The data must be moved out of the array before the loop returns to calculate the next sequence.

Any remaining questions should be referred to

Director  
Defense Nuclear Agency  
RAAE/Major Leon A. Wittwer  
Washington, DC 20305

Autovon 221-7028  
Commercial 202/325-7028

The program was programmed and debugged on a Heathkit H89  
Computer system.

# PROGRAM CIRF

```

C
C THIS PROGRAM CALCULATES RANDOM SAMPLES OF THE SCINTILLATION CHANNEL
C IMPULSE RESPONSE FUNCTION AS DEFINED IN "A TRANS-IONOSPHERIC SIGNAL
C SPECIFICATION FOR SATELLITE C-3 APPLICATIONS," (ENCLOSING DOCUMENT)
C
C THE PROGRAM INPUTS ARE
C   TAUO, SIGNAL DECORRELATION TIME(SEC)
C   FO, FREQUENCY SELECTIVE BANDWIDTH(HZ)
C   DT, TIME INCREMENT(SEC)
C       (THE TIME INCREMENT MUST BE LESS THAN OR EQUAL TO TAUO DIVIDED
C        BY TEN)
C   DD, DELAY INCREMENT(SEC)
C       (THE DELAY INCREMENT SHOULD BE LESS THAN OR EQUAL TO THE
C        RELEVANT SYSTEM SYMBOL DIVIDED BY FIVE. IF DD IS GREATER THAN
C        OR EQUAL TO SIX TENTHS DIVIDED BY FO, THEN ND(SEE BELOW) IS
C        SET TO ONE AND THE FLAT FADE SPECTRUM IS USED)
C   NT, NUMBER OF DISCRETE TIMES
C       (NT MUST BE A POWER OF TWO. NT MUST BE SUFFICIENTLY LARGE SUCH
C        THAT NT TIMES DT IS GREATER THAN OR EQUAL TO ONE HUNDRED TIMES
C        TAUO)
C   ND, NUMBER OF DISCRETE DELAYS
C       (ND MUST BE SUFFICIENTLY LARGE SUCH THAT ND TIMES DD IS GREATER
C        THAN OR EQUAL TO SIX TENTHS DIVIDED BY FO)
C   SEED, SEED FOR RANDOM NUMBER GENERATOR
C        (SEED MUST BE AN ODD REAL NUMBER)
C REQUIRED FUNCTIONS/SUBROUTINES
C   FLOAT (FIXED TO FLOATING POINT CONVERSION)
C   SETD (INITIALIZES FAST FOURIER TRANSFORM TABLE)
C   IFIX (FLOATING TO FIXED POINT CONVERSION)
C   DINT (DELAY INTEGRATOR)
C   DFAM (DF*(STANDARD DEVIATION OF FOURIER COEFFICIENT))
C   RANCOF (RANDOM DF*(FOURIER COEFFICIENT) GENERATOR)
C   PFT AND PDD (FAST FOURIER TRANSFORM)
C   POWER (SIGNAL POWER DENSITY AS A FUNCTION OF DELAY)
C   TCOR (COMPLEX SIGNAL TIME DECORRELATION FUNCTION)
C
C   DOUBLE PRECISION SEED
C
C THE FOLLOWING DIMENSION LIMITS ARE MINIMUM VALUES
C   DIMENSION H(2*NT),D(NT)
C
C   DIMENSION H(4096),D(2048)
C   COMMON /BAT/NFP,BFP,NBP,BSP,ND,DD
C   COMMON /PSD/C1,TAUO,FO
C   COMMON /RAN/SEED
C   COMMON /DFM/DFMS
C
C SET DELAY PARAMETER
C
C   DATA C1/.25/
C

```

```

C READ AND TEST DATA
C
C INPUT IS ON LOGICAL UNIT 6
C OUTPUT IS ON LOGICAL UNIT 2
C OUTPUT(CRT) IS ON LOGICAL UNIT 1
C
      CALL OPEN(6,'SY1:INPUT.DAT ')
      CALL OPEN(2,'SY1:OUTPUT.DAT ')
      READ(6,1000)TAU0,F0,DT,DD,NT,ND,SEED
1000  FORMAT(4E10.2,2I5,D10.2)
      WRITE(2,1001)TAU0,F0,DT,DD,NT,ND,SEED
1001  FORMAT(1X,1P,27HSIGNAL DECORRELATION TIME= ,E10.2/ 1X,31MFREQUENCY
1 SELECTIVE BANDWIDTH= ,E10.2/1X,16HTIME INCREMENT= ,E10.2/1X,17HDE
1LAY INCREMENT= ,E10.2/1X,17HNUMBER OF TIMES= ,15/1X,16HNUMBER OF D
1ELAYS= ,15/1X,6HSEED= ,D20.12)
      WRITE(1,1010)
1010  FORMAT(1X,3HRUNNING )
      IF(DT.LE.TAU0/10.)GO TO 10
      WRITE(2,1002)
1002  FORMAT(1X,33HERROR-TIME INCREMENT IS TOO LARGE )
      STOP
10   AMP=FLOAT(NT)
      NP=0
20   AMP=AMP/2.
      NP=NP+1
      IF(AMP.GT.1.)GO TO 20
      IF(AMP.EQ.1.)GO TO 30
      WRITE(2,1003)
1003  FORMAT(1X,30HERROR-NT IS NOT A POWER OF TWO )
      STOP
30   IF(FLOAT(NT)*DT.GE.100.*TAU0)GO TO 40
      WRITE(2,1004)
1004  FORMAT(1X,35HERROR-TIME RANGE LESS THAN 100 TAU0 )
      STOP
40   IF(DD*FLOAT(ND).GE..6/F0)GO TO 50
      WRITE(2,1005)
1005  FORMAT(1X,34HERROR-DELAY RANGE LESS THAN 0.6/F0 )
      STOP
50   IF(DD.LT..6/F0)GO TO 60
      ND=1
      WRITE(2,1006)
1006  FORMAT(1X,39HFLAY FADE LIMIT-ONLY ONE DELAY REQUIRED )
60   CONTINUE
C
C INITIALIZE FAST FOURIER TRANSFORM TABLE
C
      CALL SETD(D,NP)
      L=NT*NT-1
C
C CALCULATE FREQUENCY INCREMENT AND INTEGRATION VARIABLES
C
C DP=FREQUENCY INCREMENT

```

```

      DF=1./((FLOAT(NT)*DT)
C   NFP=NUMBER OF FREQUENCY SUBINCREMENTS
      NFP=IFIX(33.*TAUO*DF)+1
C   DFP=FREQUENCY SUBINCREMENT
      DFP=DF/FLOAT(NFP)
C   NDP=NUMBER OF DELAY SUBINCREMENTS
      NDP=IFIX(33.*DD*FO)+1
C   DDP=DELAY SUBINCREMENT
      DDP=DD/FLOAT(NDP)
C
C   CALCULATE TIME SEQUENCES FOR EACH DELAY
C
C
C   DSTART IS THE BEGINNING OF THE DELAY WINDOW. TWO PER CENT OF THE
C   SIGNAL ENERGY ARRIVES BEFORE DSTART AND THUS IS NEGLECTED WHEN DSTART
C   IS INITIALIZED TO -.04/FO.
C
      DSTART=-.04/FO
C
C   INITIALIZE VARIABLES FOR STATISTICAL TESTING. THE AVERAGE SIGNAL
C   POWER AT EACH DELAY (THE AVERAGE VARIANCE OF THE FOURIER COEFFICIENTS
C   TIMES DD**2) AND THE TIME DECORRELATION PROPERTIES OVER ALL SEQUENCES
C   NEAR TAUO WILL BE TESTED.
C
      ICOR=IFIX(TAUO/DT)
      ICOR=ICOR+ICOR+1
      CORR=0.
      COR1=0.
      DO 70 I=1,ND
      T=DSTART+.5*DD
      WRITE(1,1012)T
1012  FORMAT(1X,1P,3)NGENERATING SEQUENCE FOR DELAY= .E10.2)
      K=L
      FSTART=.5*DD
      AMP=1.
C
C   ZERO DC FREQUENCY COMPONENT TO MAINTAIN ZERO MEAN SEQUENCE
C
      H(1)=0.
      H(2)=0.
C
C   DFM5=INITIAL DELAY INTEGRATION POINT FOR DFM
C
      DFM5=.5*PINT(DSTART,FSTART)
      DO 80 J=3,NT,2
      IF(AMP.LE.0.100 TO 85
      AMP=(FAM(DSTART,FSTART)
      CALL RANCOF(N(I),N(I+1),AMP)
      CALL RANCOF(N(I),N(I+1),AMP)
      FSTART=FSTART+DD
      GO TO 80
85  NIJ)=0.

```



```

      H(J+1)=0.
      H(K)=0.
      H(K+1)=0.
80    K=K-2
      AMP=DFAM(DSTART,FSTART)
      CALL RANCOF(H(K),H(K+1),AMP)
C
C USE FAST FOURIER TRANSFORM TO GENERATE FINAL SEQUENCE
C
      CALL PFT(H,D)
      CALL PDD(H)
C
C TEST SIGNAL POWER IN SEQUENCE TERMS AND ACCUMULATE DATA FOR TIME
C CORRELATION TEST. USE TRAPEZOIDAL INTEGRATION.
C
      TVAR=0.
      DO 90 J=ICOR,L+2
      K=J-ICOR+1
      CORR=CORR+H(K)*H(J)+H(K+1)*H(J+1)
      CORI=CORI+H(K+1)*H(J)-H(K)*H(J+1)
90    TVAR=TVAR+H(J)*H(J)+H(J+1)*H(J+1)
      AMP=2.0*TVAR*DD**2/FLOAT(L-ICOR+2)
      PSUM=.5*POWER(DSTART)
      T=DSTART
      DO 100 J=1,NDP
      Y=T+DDP
      SM=POWER(Y)
100    PSUM=PSUM+SM
      PSUM=(PSUM-.5*SM)*DDP
      T=DSTART+.5*DD
      WRITE(2,1000)T,AMP,PSUM
1000  FORMAT(1X,1P,7HDELAY= .E10.2/1X,24HAVERAGE SEQUENCE POWER= .
      1E10.2/1X,29HTHEORETICAL SEQUENCE POWER= .E10.2)
C
C THE H ARRAY NOW CONTAINS A SAMPLE DIGITAL REPRESENTATION OF THE
C CHANNEL IMPULSE RESPONSE FUNCTION FOR A DELAY OF DSTART+.5*DD. THE
C IN-PHASE AND QUADRATURE MULTIPLIERS (THE REAL AND IMAGINARY PARTS OF
C THE CHANNEL IMPULSE RESPONSE FUNCTION, RESPECTIVELY) ARE DEFINED BY
C
C      IN-PHASE COMPONENT MULTIPLIER = H(I+I-1)
C      QUADRATURE COMPONENT MULTIPLIER = H(I+I)
C
C WHERE I INDEXES SUCCESSIVE VALUES IN TIME. THE USER MUST DECIDE ON
C THE DISPOSITION OF THE DATA IN THE H ARRAY BEFORE PROCEEDING ON TO
C THE NEXT DELAY. USUALLY, THE DATA IS STORED AND LATER USED IN AN
C INTEGRAL OVER DELAY. IN THESE INTEGRATIONS, DD MUST BE USED
C EXPLICITLY AS THE INTEGRATION INCREMENT.
C
70    DSTART=DSTART+DD
C
C TEST TIME CORRELATION PROPERTIES
C

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CORR=2.0*CORR*DD**2/FLOAT(L-ICOR+2)
CORI=2.0*CORI*DD**2/FLOAT(L-ICOR+2)
T=.5*FLOAT(ICOR-1)*DT
AMP=TCOR(T)
WRITE(2,1009)T,CORR,CORI,AMP
1009 FORMAT(1X,1P,25HTIME DISPLACEMENT TESTED= ,E10.2/ 1X,26HCALCULATE
1D DECORRELATION= ,2E10.2/1X,25HTHEORETICAL CALCULATION= ,E10.2)
WRITE(2,1007)
1007 FORMAT(1X,10HEND OF RUN )
END
FUNCTION DFAM(DSTART,FSTART)
C
C THIS FUNCTION CALCULATES DF TIMES THE STANDARD DEVIATION OF THE RANDOM
C FOURIER COEFFICIENT CORRESPONDING TO A DELAY OF DSTART+.5*DD AND A
C FREQUENCY OF FSTART+.5*DF. THE VARIANCE OF THIS FOURIER COEFFICIENT
C IS THE GENERALIZED POWER SPECTRAL DENSITY INTEGRATED OVER DELAY FROM
C DSTART TO DSTART+DD AND OVER FREQUENCY FROM FSTART TO FSTART+DF.
C INTEGRATION IS BY THE TRAPEZOIDAL RULE.
C
C ARGUMENTS
C DSTART=INITIAL DELAY(SEC)
C FSTART=INITIAL FREQUENCY(HZ)
C REQUIRED COMMON VARIABLES
C NFP=NUMBER OF FREQUENCY SUBINCREMENTS
C DFP=FREQUENCY SUBINCREMENT(HZ)
C DDP=DELAY SUBINCREMENT(SEC)
C DD=DELAY INCREMENT
C DFMS=.5*DINT(DSTART,FSTART)
C FUNCTIONS REQUIRED
C DINT (DELAY INTEGRATOR)
C SORT (FLOATING POINT SQUARE ROOT)
C
COMMON /DST/NFP,DFP,DDP,DD,DFMS
COMMON /DFN/DFMS
F=FSTART
DFAM=DFMS
DO 10 I=1,NFP
F=F+DFP
DM=DINT(DSTART,F)
10 DFAM=DFAM*DM
DFMS=.5*DM
DFAM=SQRT(DDP*DDP*(DFAM-DFMS))/DD
RETURN
END
FUNCTION DINT(DSTART,F)
C
C THIS FUNCTION INTEGRATES THE GENERALIZED POWER SPECTRUM FROM A DELAY
C OF DSTART TO DSTART+DD AT A FREQUENCY OF F AND DIVIDES THE RESULT BY
C DDP.
C
C ARGUMENTS
C DSTART=INITIAL DELAY(SEC)

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C F=FREQUENCY(HZ)
C REQUIRED COMMON VARIABLES
C ND=FLAT FADE FLAG
C ND=1, FLAT FADING
C ND=0, 1, FREQUENCY SELECTIVE FADING
C NDP=NUMBER OF DELAY SUBINCREMENTS
C DDP=DELAY SUBINCREMENT
C FUNCTIONS REQUIRED
C PSD3 (GENERALIZED POWER SPECTRUM)
C PSDFF (FLAT FADE SPECTRUM)
C
COMMON /DAT/NFP,DFP,NDP,DDP,ND,DD
C
C TEST FOR FLAT FADING
C
IF(ND.EQ.1)GO TO 100
D=DSTART
DINT=.5*PSD3(F,D)
DO 10 I=1,NDP
D=D+DDP
SM=PSD3(F,D)
10 DINT=DINT+SM
DINT=DINT-.5*SM
RETURN
100 DINT=PSDFF(F)/DDP
RETURN
END
FUNCTION PSD3(F,D)
C
C THIS FUNCTION IS THE GENERALIZED POWER SPECTRUM FOR FREQUENCY
C SELECTIVE CASES.
C
C ARGUMENTS
C F=FREQUENCY(HZ)
C D=DELAY(SEC)
C REQUIRED COMMON VARIABLES
C C1=DELAY PARAMETER
C TAU=SIGNAL DECORRELATION TIME
C F0=FREQUENCY SELECTIVE BANDWIDTH
C FUNCTIONS REQUIRED
C SQRT (FLOATING POINT SQUARE ROOT)
C ABS (FLOATING POINT ABSOLUTE VALUE)
C EXP (FLOATING POINT NATURAL EXPONENTIAL)
C
COMMON /PS3/C1,TAU,F0
FPRIME=F0+SQRT(1.+C1**2)
TFPPD=6.283*FPRIME*D
Z=.7071*(C1+(13.142*TAU**2)**2-TFPPD/C1)
IF(ABS(Z).LT.1)GO TO 10
Z=1./Z**2
IF(Z.LT.0)GO TO 20
QZ=1.259*EXP(-Z**2)*(1.+(1.197-.112*Z)+Z-.191)*Z)/SQRT(Z)

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      GO TO 50
20    OZ=1.772*(1.+(1.0014*X-.179)*X+.29)*X)/SQRT(-Z)
      GO TO 50
10    OZ=((1.031*Z-.109)*Z-.729)*Z-.675)*Z
      OZ=1.313*EXP(OZ)
50    PSD3=2.981*FPRIME*TAUO*EXP(.5*C1**2-TPFPD)*OZ/SQRT(C1)
      RETURN
      END
      FUNCTION PSDFF(F)

C
C THIS FUNCTION IS THE FLAT FADE POWER SPECTRUM. THIS FUNCTION IS NOT
C THE GENERALIZED POWER SPECTRUM(FUNCTION PSD3) INTEGRATED OVER ALL
C DELAY. A CLOSE APPROXIMATION HAS BEEN CHOSEN INSTEAD BECAUSE OF ITS
C UTILITY IN SOFTWARE AND HARDWARE APPLICATIONS. WHITE NOISE FILTERED
C BY TWO SINGLE POLE FILTERS REPRODUCES THIS SPECTRUM.
C
C ARGUMENTS
C F=FREQUENCY(HZ)
C REQUIRED COMMON VARIABLES
C TAUO=SIGNAL DECORRELATION TIME
C
      COMMON /PSD/C1,TAUO,F0
      PSDFF=1.864*TAUO/(1.+9.572*(F*TAUO)**2)**2
      RETURN
      END
      SUBROUTINE RANCOF(XR,XI,AMP)

C
C THIS FUNCTION RETURNS A COMPLEX RANDOM NUMBER. (XR,XI), WHERE THE REAL
C AND IMAGINARY PARTS ARE INDEPENDENT ZERO MEAN NORMALLY DISTRIBUTED
C RANDOM VARIABLES EACH WITH A VARIANCE OF .5*AMP**2.
C
C ARGUMENTS
C XR=RETURNED REAL RANDOM SAMPLE
C XI=RETURNED IMAGINARY RANDOM SAMPLE
C AMP=SQUARE ROOT OF TWO TIMES THE VARIANCE OF XR AND XI
C REQUIRED FUNCTIONS
C SORT (FLOATING POINT SQUARE ROOT)
C ALOG (FLOATING POINT NATURAL LOG)
C RAND (SAMPLE OF UNIFORM DISTRIBUTION BETWEEN ZERO AND ONE)
C COS (FLOATING POINT COSINE)
C SIN (FLOATING POINT SINE)
C
      AM=AMP*SQRT(-ALOG(RAND(0)))
      AN=6.283185*RAND(0)
      XR=AM*COS(AN)
      XI=AM*SIN(AN)
      RETURN
      END
      FUNCTION RAND(I)

C
C THIS FUNCTION GENERATES RANDOM NUMBERS UNIFORMLY DISTRIBUTED
C (APPROXIMATELY) IN THE INTERVAL OF ZERO TO ONE. THE USER SHOULD

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C REPLACE THIS ALGORITHM WITH THE BETTER ONE (PRESUMABLY) ON HIS OWN
C SYSTEM.
C
C ARGUMENTS
C I=DUMMY ARGUMENT
C REQUIRED COMMON VARIABLES
C SEED=RANDOM NUMBER SEED (INITIAL VALUE MUST BE ODD)
C REQUIRED FUNCTIONS
C DMOD (DOUBLE PRECISION MOD FUNCTION)
C
      DOUBLE PRECISION SEED
      COMMON /RAN/SEED
      SEED=DMOD(4.8939125D+07*SEED,2.68435456D+08)
      RAND=(SEED/2.68435457D+08)
      END
      FUNCTION POWER(D)
C
C THIS FUNCTION CALCULATES THE GENERALIZED POWER SPECTRUM AT A DELAY OF
C D INTEGRATED OVER ALL FREQUENCY.
C
C ARGUMENTS
C D=DELAY (SEC)
C REQUIRED COMMON VARIABLES
C C1=DELAY PARAMETER
C F0=FREQUENCY SELECTIVE BANDWIDTH
C REQUIRED FUNCTIONS
C SORT (FLOATING POINT SQUARE ROOT)
C ABS (FLOATING POINT ABSOLUTE VALUE)
C EXP (FLOATING POINT NATURAL EXPONENTIAL)
C
      COMMON /PSB/C1,TAU0,F0
      FPRIME=F0*SORT(1.+C1**2)
      TFFPD=0.293*FPRIME*D
      Z=.7071*(C1-TFFPD/C1)
      T=1./1.+*.47047*ABS(Z)
      FT=(1.7479556*T-.0959799)*T+.3490242)*T
      IF(Z.LT.0.100 TO 10
      POWER=3.1416*FPRIME*EXP(-.5*(TFFPD/C1)**2)*FT
      RETURN
10  POWER=3.1416*FPRIME*EXP(.5*(C1**2-TFFPD))*(2.-FT*EXP(-Z**2))
      RETURN
      END
      FUNCTION TCOR(T)
C
C THIS FUNCTION CALCULATES THE COMPLEX SIGNAL TIME DECORRELATION
C FUNCTION.
C
C ARGUMENTS
C T=TIME DISPLACEMENT (SEC)
C REQUIRED COMMON VARIABLES
C TAU0=SIGNAL DECORRELATION TIME (SEC)
C ND=FLAT FACE FLAG

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C ND=1, FLAT FADING
C ND.GT.1, FREQUENCY SELECTIVE FADING
C REQUIRED FUNCTIONS
C EXP (FLOATING POINT NATURAL EXPONENTIAL)
C ABS (FLOATING POINT ABSOLUTE VALUE)
C
C      COMMON /PSD/C1,TAU0,F0
C      COMMON /DAT/NFP,DFP,NDF,D:P,ND,DB
C
C CHECK FOR FLAT FADING
C
C      IF(ND.EQ.1)GO TO 10
C      TCR=EXP(-(T/TAU0)**2)
C      RETURN
10  TCR=2.146*ABS(T)/TAU0
C      TCR=EXP(-TCR)*(1.+TCR)
C      RETURN
C      END
C      SUBROUTINE SETD(TABL,NT)
C
C SUBROUTINES SETD, PFT, AND PDD ACCOMPLISH THE FAST FOURIER TRANSFORM
C ON A COMPLEX ARRAY,C. CR AND CI WILL BE USED AS A SHORT HAND TO
C REPRESENT THE REAL AND IMAGINARY PARTS OF C. LET
C      CR(I) = C(I+1)
C      CI(I) = C(I+I)
C DEFINE THE CORRESPONDENCE BETWEEN C, CR, AND CI. THE RESULTS ARE
C RETURNED IN THE C ARRAY. THE NUMBER OF POINTS TO BE TRANSFORMED IS
C NP=2*NT. THE FAST FOURIER TRANSFORM REQUIRES A TABLE REPRESENTED BY
C THE COMPLEX TABL ARRAY. MINIMUM LIMITS FOR THE C AND TABL ARRAYS ARE
C
C      DIMENSION C(NP+NP),TABL(NP)
C
C (CR(I),CI(I)) AT THE OUTPUT EQUALS THE SUM OVER J FROM 1 TO NP OF
C
C (CR(I)*COS(2.*PI*M*N/NP)-CI(I)*SIN(2.*PI*M*N/NP),CR(I)*SIN(2.*PI*M*N/
C NP)+CI(I)*COS(2.*PI*M*N/NP))
C
C WHERE THE COMPLEX NOTATION HAS BEEN USED. (REAL,IMAGINARY), AND
C WHERE N=J-1 WHEN J.GE.1.AND.J.LE.NP/2+1
C N=J-NP-1 WHEN J.GE.NP/2+1.AND.J.LE.NP
C M=1-1 WHEN 1.GE.1.AND.1.LE.NP/2+1
C M=1-NP-1 WHEN 1.GE.NP/2+1.AND.1.LE.NP
C
C SUBROUTINE SETD INITIALIZES THE TABL ARRAY AND NEED BE CALLED ONLY
C WHEN NT IS CHANGED. SUBROUTINES PFT AND PDD WHEN CALLED IN SEQUENCE
C ACCOMPLISH THE TRANSFORM.
C
C      CALL SETD(TABL,NT)
C
C      CALL PFT(C,TABL)
C      CALL PDD(C)
C

```

C IN THE ABOVE SUM, M IS PROPORTIONAL TO THE VARIABLE REPRESENTED BY THE  
 C I INDEX. FOR EXAMPLE, IF (CR(I),CI(I)) IS A FREQUENCY COEFFICIENT,  
 C THEN ZERO FREQUENCY IS REPRESENTED BY M EQUAL TO ZERO AND I EQUAL TO  
 C ONE. SIMILARLY, N IS PROPORTIONAL TO THE VARIABLE REPRESENTED BY THE  
 C J INDEX.

C  
 C FAST FOURIER TRANSFORMS ASSUME CONTINUITY OF THE FUNCTION TO BE  
 C TRANSFORMED AND THE RESULT AT I (OR J) EQUAL TO NP/2+1 WHICH REPRESENTS  
 C THE EXTREMES OF M (OR N). THE FUNCTIONS ARE ALSO CONTINUOUS BETWEEN  
 C (CR(1),CI(1)) AND (CR(NP),CI(NP)).

C  
 C REQUIRED FUNCTIONS  
 C FLOAT (FIXED TO FLOATING POINT CONVERSION)  
 C MOD (FIXED POINT MOD)  
 C COS (FLOATING POINT COSINE)  
 C SIN (FLOATING POINT SINE)

C  
 C \*\*\*\*\*  
 C THE ARRAYS IN SETD,PFT, AND PDD HAVE BEEN DIMENSIONED TO ONE  
 C INTERNALLY. THIS MAY NOT BE ALLOWED ON THE USER SYSTEM. IT MAY BE  
 C NECESSARY TO DIMENSION TO THE LIMITS IN THE MAIN PROGRAM OR TO USE  
 C VARIABLE ARRAY DECLARATORS.

C \*\*\*\*\*  
 C

```

      DIMENSION TABL(1)
      COMMON /FFT/N2,NTI,NL
      DATA PI/3.1415926536/
      NTI=NT
      N2=2**NT
      NL=N2-1
      NN=N2/2-1
      TPI=2.*PI/FLOAT(N2)
      TABL(1)=1.
      TABL(2)=0.
      DO 40 I=1,NN
        J=I+I
        II=0
      DO 20 N=1,NT
        II=II+II+MOD(J,2)
20      J=J/2
        K=I+I+1
        TABL(K)=COS(TPI*FLOAT(II))
40      TABL(K+1)=SIN(TPI*FLOAT(II))
      RETURN
      END
      SUBROUTINE PFT(C,TABL)
      DIMENSION C(1),TABL(1)
      COMMON /FFT/N2P,NTI,NL
      N1=2
      N2=N2P
      DO 1 I=1,NTI
        NN=-N2-N2

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      DO 2 J=2,N1,2
      NN=NN+N2+N2
      N3=NN+N2
      DO 3 K=2,N2,2
      N4=N3+K
      STR=C(N4-1)*TABL(J-1)-C(N4)*TABL(J)
      STI=C(N4-1)*TABL(J)+C(N4)*TABL(J-1)
      N5=NN+K
      C(N4-1)=C(N5-1)-STR
      C(N4)=C(N5)-STI
      C(N5-1)=C(N5-1)+STR
3     C(N5)=C(N5)+STI
2     CONTINUE
      N1=N1+N1
1     N2=N2/2
      RETURN
      END
      SUBROUTINE PDD(C)
C
C   REQUIRED FUNCTIONS
C   MOD (FIXED POINT MOD)
C
      DIMENSION C(1)
      COMMON /FFT/N2,NTI,NL
      DO 1 I=1,NL
      J=I
      II=0
      DO 3 N=1,NTI
      II=II+II+MOD(J,2)
3     J=J/2
      IF(II-I)1,1,7
7     IN=I+I+1
      IIN=II+II+1
      DR=C(IN)
      DI=C(IN+1)
      C(IN)=C(IIN)
      C(IN+1)=C(IIN+1)
      C(IIN)=DR
      C(IIN+1)=DI
1     CONTINUE
      RETURN
      END

```



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